Anomalous Beta Spectrum of In^{114} ($1^+ \to 0^+$)

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We analyze the experimental beta-shape factor of In^{114} ($1^+ \to 0^+$) as reported to be represented by $(1+b/W)$. We find a reasonable fit to the experimental shape factor of the Langer group by considering the contribution of the second-order effects within the framework of the V-A theory. The less anomalous beta-shape factor due to Daniel and Panussi can also be explained. The effects arising from a consideration of the finite nuclear size and the finite de Broglie wavelength were included in our calculations.

1. INTRODUCTION

SEVERAL beta-shape factors of the allowed transitions have been reported[1] as being represented by $(1+b/W)$. The values of the parameter b reported by the Langer group[1] lie in the range

$$0.2<b<0.3, \tag{1a}$$

whereas an independent measurement of the beta-shape factor of In^{114} by Daniel and Panussi[2] is reported as leading to

$$0.03\leq b\leq 0.07. \tag{1b}$$

While the latter measurement is clearly not compatible with the results of the Langer group, it is interesting to note that both these measurements indicate the same form of the energy dependence. Normally, one would expect the value of b to be essentially zero. Consequently, there have been several attempts, particularly by Eman and Tadic[3] and Pearson,[4] in finding suitable explanations of the anomalous beta shape factor. Eman and Tadic calculate the beta-shape factor formula with the conventional β-decay theory taking into account the corrections arising from the weak magnetism term and the induced pseudoscalar interaction. However, they make many simplifying assumptions in the actual analysis of the experimental shape factor. On the other hand, Pearson finds a reasonable fit to the data of Langer *et al.* on the basis of a very large contribution of the induced pseudoscalar interaction.[5] In both of these analyses, approximate functions occurring in the theoretical formulas were employed, i.e., expansions in the power of $\alpha Z/p$.

It is the purpose of this paper to examine whether or not the energy dependence factor in the experimental beta-shape factor of In^{114} ($1^+ \to 0^+$) can be explained within the framework of the V-A theory. In addition, we analyze the beta-longitudinal-polarization measurement of Spivak *et al.*,[6] which gives

$$P_{11}=(-0.93\pm0.06)v/c \tag{2}$$

for beta momentum equal to 1.23.

2. THEORY

The beta-shape factor and the beta-longitudinal polarization calculated from the standard V-A theory for ($1^+ \to 0^+$) transitions are given by Eq. (3) and Eq. (4), respectively.[7]

$$C=\left|\int\boldsymbol{\sigma}\right|^2 g_A{}^2[b_0+(b_1+b_2\eta)\xi_1+b_3\xi_2] \tag{3}$$

$$-\frac{P_{11}}{v/c}=\frac{a_0+(a_1+a_2\eta)\xi_1+a_3\xi_2}{b_0+(b_1+b_2\eta)\xi_1+b_3\xi_2}, \tag{4}$$

where we use the following notation[8]:

$$b_0=L_0,\qquad b_1=-(\tfrac{1}{3}q^2L_0+\tfrac{2}{3}qN_0)$$
$$b_2=-\tfrac{4}{3}qN_0,\qquad b_3=(g_V/g_A)\times2(N_0+\tfrac{1}{3}qL_0) \tag{5}$$

$$a_0=-A_0,\qquad a_1=\tfrac{1}{3}q^2A_0+\tfrac{1}{3}qD_0$$
$$a_2=-\tfrac{2}{3}qD_0,\qquad a_3=-(g_V/g_A)(D_0+\tfrac{2}{3}qA_0) \tag{6}$$

$$\xi_1\equiv\int\boldsymbol{\sigma}r^2\Big/\int\boldsymbol{\sigma},\qquad \xi_2\equiv\left(\frac{1}{M}\right)\left(\frac{3}{2}\right)^{1/2}\int\mathbf{r}\times\mathbf{p}\Big/\int\boldsymbol{\sigma},$$
$$=\int\boldsymbol{\alpha}\times\mathbf{r}\Big/\int\boldsymbol{\sigma} \tag{7}$$

TABLE I. In^{114} ($1^+ \to 0^+$). Numerical coefficients for beta longitudinal polarization and shape-factor formulas.[a]

p	a_0	a_1	a_2	a_3	b_0	b_1	b_2	b_3
0.4	0.938	19.55	48.11	14.00	0.935	18.81	46.60	13.50
0.6	0.937	19.25	47.08	14.07	0.935	18.57	45.68	13.61
1.0	0.934	18.36	44.16	14.30	0.932	17.84	43.11	13.92
1.2	0.933	17.79	42.38	14.42	0.931	17.30	41.88	14.10
2.0	0.925	14.99	34.26	15.02	0.924	14.74	33.76	14.79
3.0	0.915	10.43	22.64	15.83	0.914	10.33	20.65	15.68
4.0	0.903	4.87	10.08	16.65	0.904	4.84	9.68	16.53

[a] Equations (3) and (4). These coefficients have been calculated considering (1) the nuclear radius to be $0.428\,\alpha A^{1/3}$ F, (2) the corrections due to the finite nuclear size, and (3) the finite de Broglie wavelength effects.

[1] O. E. Johnson, R. G. Johnson, and L. M. Langer, Phys. Rev. **112**, 2004 (1958); J. H. Hamilton, L. M. Langer, and W. G. Smith, *ibid.* **112**, 2010 (1958); **119**, 772 (1960).
[2] H. Daniel and Ph. Panussi, Z. Physik **164**, 303 (1961).
[3] B. Eman and D. Tadic, Glansik Mat.-Fiz. Astron. Ser. II: **16**, 89 (1961).
[4] J. M. Pearson, Phys. Rev. **126**, 1100 (1962).
[5] This assumed contribution of the induced P interaction is much greater than theoretically predicted. For example, see M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
[6] P. E. Spivak, L. A. Mikaéljan, I. E. Kutikov, and V. F. Analin, J. Exptl. Theoret. Phys. **39**, 1477 (1960) [translation: Soviet Phys.—JETP **12**, 1027 (1961)].
[7] See, for example, M. Morita, and R. S. Morita, Phys. Rev.

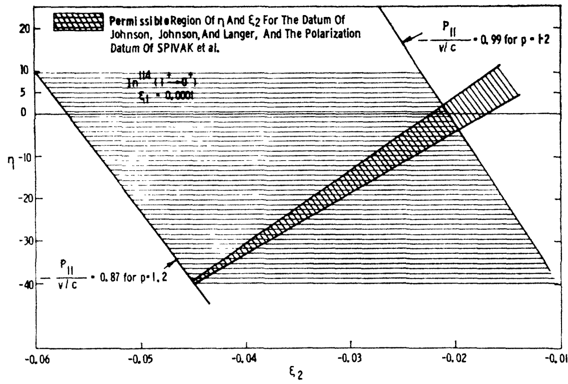

FIG. 1. The permissible value of η and ξ_2, the ratios of the nuclear matrix elements defined in Eq. (7), for a reasonable fit to the experimental shape factor of the Langer group and the beta polarization data of Spivak *et al.* (reference 6).

$$\eta\equiv-\int\boldsymbol{\sigma}\cdot\mathbf{r}\mathbf{r}\bigg/\int\boldsymbol{\sigma}r^2,$$

$$q=(W_0-W),\qquad W=(p^2+1)^{1/2}.\qquad(8)$$

$$L_0=(g_{-1}^2+f_1^2)(2p^2F)^{-1},$$

$$N_0=(f_{-1}g_{-1}-f_1g_1)(2p^2F\rho)^{-1},$$

$$A_0=f_1g_{-1}\sin(\delta_1-\delta_{-1})(p^2F)^{-1}W/p,$$

$$D_0=(f_1f_{-1}-g_1g_{-1})\sin(\delta_1-\delta_{-1})(p^2F\rho)^{-1}W/p.$$

F is the Fermi function and the nuclear radius ρ is in units of $\hbar/mc$. W_0, the end-point energy (mc^2 units), is 4.87 for In^{114} $(1^+\rightarrow 0^+)$. We assume the validity of the two-component neutrino theory and the time-reversal invariance in the strong and the weak interaction. It should be noted that all the terms in Eqs. (3) and (4), except for b_0 and a_0, arise because of the second-order effects, e.g., $a_3\xi_2$ and $b_3\xi_2$ terms are the contribution of the vector interaction. We do not consider the interference terms between the forbidden matrix elements.

In Table I, we give the electronic functions for the beta-shape factor and the beta polarizations in Eq. (5) and Eq. (6) for In^{114}. These electronic functions were computed from the tables of Bhalla and Rose[9] who have included the finite nuclear size effects[10] and the finite de Broglie wavelength effects.[11]

3. NUMERICAL RESULTS

Since it is not clear which of the two measurements[1,2] represents the true beta-shape factor, we carried out extensive analyses of the two measurements. First, we present our numerical results for the data of Johnson, Johnson and Langer.

A. Data of Johnson, Johnson and Langer (reference 1)

In our analysis we consider the ratios of the nuclear matrix elements, i.e., ξ_1, ξ_2, and η as parameters. For simplicity,[12] we have taken

$$\xi_1=0.0001.\qquad(10)$$

The admissible values of ξ_2 and η are shown in Fig. 1. The experimental beta-shape factor of Langer *et al.* is easily explained for the values of ξ_2 and η shown by the oblique shaded region. The horizontal shaded region corresponds to the beta polarization (in units of v/c) at $p=1.23$. We have taken the calculated shape factor to be a reasonable fit if

$$\bar{\Delta}\leq 0.0004,\qquad(11)$$

where

$$\bar{\Delta}=(1/7)\sum_{i=1}^{8}(\Delta X_i/X_i)^2.\qquad(12)$$

In Eq. (12), ΔX_i is the difference between the calculated shape factor from the corresponding X_i given by the $(1+0.25/W)$ curve. We have taken eight values of beta momentum at equal intervals of $p=0.5$ starting from $p=4.0$. The maximum value of $\Delta X_i/X_i$ is generally less than 5% for $\bar{\Delta}=0.0004$.

In Fig. 2, the calculated beta-shape factors are plotted for various values of η and ξ_2 keeping $\xi_1=0.0001$.

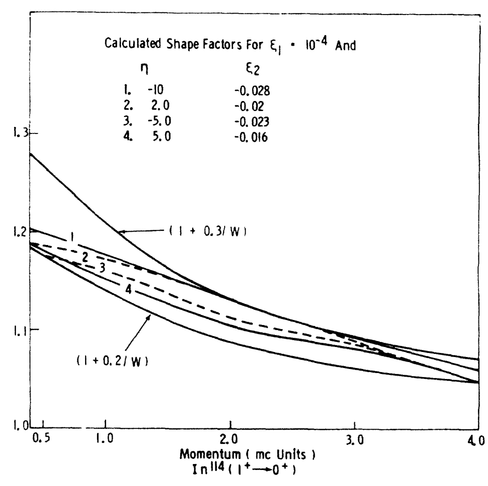

FIG. 2. Calculated beta-shape factors for various values of the ratios of the nuclear matrix elements. The experimental data of reference 1 is represented by $1+b/W$ for $0.2<b<0.3$.

109, 2048 (1958); M. Morita, *ibid.* **113**, 1584 (1959); and B. Eman and D. Tadic, reference 3.

[8] C. P. Bhalla and M. E. Rose, Phys. Rev. **120**, 1415 (1960); M. E. Rose and R. K. Osborn, *ibid.* **93**, 1315 (1954).

[9] C. P. Bhalla and M. E. Rose, Oak Ridge National Laboratory Report, ORNL-3207, 1962 (unpublished).

[10] See, for example, M. E. Rose and D. K. Holmes, Phys. Rev. **83**, 190 (1951); C. P. Bhalla and M. E. Rose, *ibid.* **128**, 774 (1962).

[11] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953).

[12] Since the theoretical values of ξ_1 (and other ratios of the nuclear matrix elements) cannot be calculated with complete confidence, we have chosen the particular value of ξ_1, which is of the correct order of magnitude as calculated from Morita's method (reference 7). However, this choice does not put any restrictions on the conclusions of our analysis because one can find reasonable fits to the experimental data (reference 1) for other values of ξ_1.

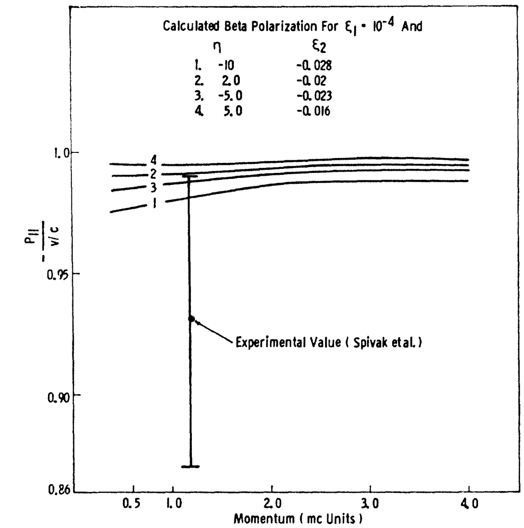

FIG. 3. In^{114} $(1^+ \rightarrow 0^+)$. The beta-longitudinal polarization (in units of $-v/c$) for various values of the ratios of the nuclear matrix elements vs beta momentum. The experimental value (0.93 ± 0.06) is also shown.

The experimental shape factor[1] is also shown with the upper and lower limits as $(1+0.3/W)$ and $(1+0.2/W)$, respectively. The corresponding calculated beta polarization versus beta momentum is given in Fig. 3.

B. Data of Daniel and Panussi (reference 2)

The results of our analysis are given in Fig. 4. The shaded area in Fig. 4 represents the permissible values of the ratios of the nuclear matrix elements for $\bar{\Delta} \leq 0.0002$. We obtained $\bar{\Delta}$, as defined in Eq. (12), by computing ΔX_i as the difference between the calculated shape factor and the curve $(1+0.05/W)$. As in Sec. 3(A), we find that the beta-longitudinal-polarization measurement of Spivak *et al.*[6] can also be explained.

4. CONCLUSIONS

Previous attempts in obtaining a reasonable fit to the data of the Langer group[1] were unsuccessful within the framework of the standard V-A theory. From the analysis presented here, it is clear that the shape factor of In^{114} $(1^+ \rightarrow 0^+)$, as reported by the Langer group, can be easily explained by the conventional V-A interaction. It should be noted that the experimental beta polarization of Spivak *et al.* can be also fitted with the same values of the nuclear matrix elements without any contribution of the induced pseudoscalar interaction. The reason for our excellent agreement between the calculated and the experimental shape factor is the use of accurate electronic radial functions in our extensive analysis and a consideration of the second-order effects. These considerations are of significant importance whenever there is a large destructive interference, as in the present case.

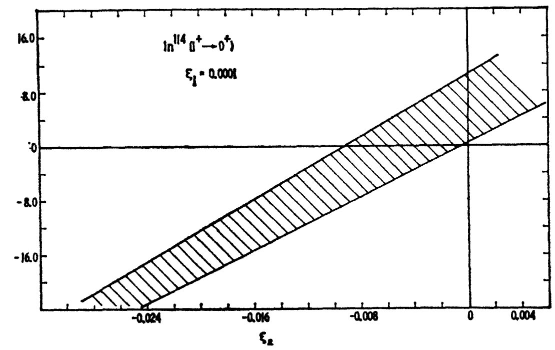

FIG. 4. The permissible values of η and ξ_2 for a reasonable fit to the beta-shape factor of In^{114} $(1^+ \rightarrow 0^+)$ as given by Daniel and Panussi (reference 2).

If, on the other hand, the measurement of Daniel and Panussi[2] represents the true beta-shape factor of In^{114} $(1^+ \rightarrow 0^+)$, it implies that the shape factor has essentially a (typically) allowed shape because of the smallness of b. A rather small contribution of the second-order effects was sufficient to explain the $(1+b/W)$ dependence. This is not surprising, because of the very weak energy dependence of the measured beta-shape factor.[2]

Since no other independent (and accurate) measurements of the beta-shape factor of In^{114} $(1^+ \rightarrow 0^+)$ exist except the two discussed in this paper, it is suggested that further experimental work be done in this direction. Furthermore, an accurate measurement of the beta-longitudinal polarization is also desirable.

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